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Surname **MODEL SOLUTIONS**	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced
Paper 2: Pure Mathematics 2

Specimen Paper **Time: 2 hours**	Paper Reference **9MA0/02**

You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1.

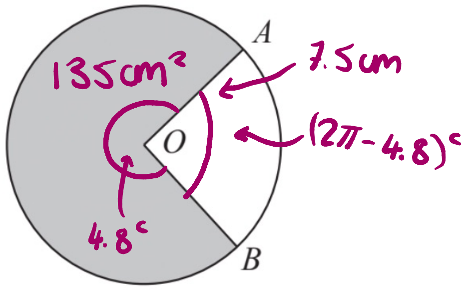

Figure 1

Figure 1 shows a circle with centre O. The points A and B lie on the circumference of the circle.

The area of the major sector, shown shaded in Figure 1, is $135\,\text{cm}^2$.

The reflex angle AOB is 4.8 radians.

Find the exact length, in cm, of the minor arc AB, giving your answer in the form $a\pi + b$, where a and b are integers to be found.

(4)

Arc length: $l = r\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$

$135 = \frac{1}{2}r^2 \times 4.8$ (1)

$28.125 = \frac{1}{2}r^2$

$56.25 = r^2$

$r = 7.5\text{ cm}$ (1)

length of the minor arc

$l = 7.5(2\pi - 4.8)$ (1)

$= 15\pi - 36\text{ cm}$ (1)

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2. (a) Given that θ is small, use the small angle approximation for $\cos\theta$ to show that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$$

(3)

Adele uses $\theta = 5°$ to test the approximation in part (a).

Adele's working is shown below.

Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$

(b) (i) Identify the mistake made by Adele in her working.

(ii) Show that $8 - 5\theta^2$ can be used to give a good approximation to $1 + 4\cos\theta + 3\cos^2\theta$ for an angle of size 5°

(2)

a) For small angle θ, measured in radians $\cos\theta \approx 1 - \frac{\theta^2}{2}$ ①

$1 + 4\cos\theta + 3\cos^2\theta \approx 1 + 4\left(1 - \frac{\theta^2}{2}\right) + 3\left(1 - \frac{\theta^2}{2}\right)^2$

$= 1 + 4 - \frac{4\theta^2}{2} + 3\left(1 - \frac{\theta^2}{2} - \frac{\theta^2}{2} + \frac{\theta^4}{4}\right)$ ①

$= 5 - 2\theta^2 + 3 - 3\theta^2 + \frac{3\theta^4}{4}$

$= 8 - 5\theta^2 + \cancel{\frac{3\theta^4}{4}}$ ← Ignore high power of θ

$\therefore 1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ ①

b i) Adele's working is degrees not radians ①

b ii) $\div 36 \left(\begin{matrix} 180° = \pi^c \\ 5° = \frac{\pi}{36}^c \end{matrix} \right) \div 36$

$8 - 5\left(\frac{\pi}{36}\right)^2 = 7.962$ $\therefore \theta = 5°$ gives a good approximation ①

3. A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ °C, is modelled by the equation

$$\theta = 25 + Ae^{-0.03t}$$

where A is a constant.

$\to t=0$

The temperature of the tea was 75 °C when the cup was placed on the table.

(a) Find a complete equation for the model. **(1)**

$t=0$

(b) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place. **(2)**

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(c) evaluate the model, explaining your reasoning. **(1)**

a) $\theta = 75 \quad t=0 \quad \therefore \quad 75 = 25 + Ae^{-0.03(0)}$

$75 = 25 + A$

$A = 50$

$\theta = 25 + 50e^{-0.03t}$ 1

b) $60 = 25 + 50e^{-0.03t}$

$35 = 50e^{-0.03t}$

$e^{-0.03t} = 0.7$ 1

$\ln e^{-0.03t} = \ln 0.7$

$\ln x^n = n \ln x$

$-0.03t \ln e = \ln 0.7$

$\ln e = 1$

$-0.03t = \ln 0.7$

$t = \frac{\ln 0.7}{-0.03} = 11.889 = 11.9$ minutes 1

2 hours = 120 minutes

c) when $t = 120$

$\theta = 25 + 50e^{-0.03(120)}$

$= 26.366$ 3dp

26.366 is not aproximately equal to 20.3, so the model is not true for lage values of t 1

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4. (a) Sketch the graph with equation

$$y = |2x - 5|$$

stating the coordinates of any points where the graph cuts or meets the coordinate axes.
(2)

(b) Find the values of x which satisfy

$$|2x - 5| > 7$$

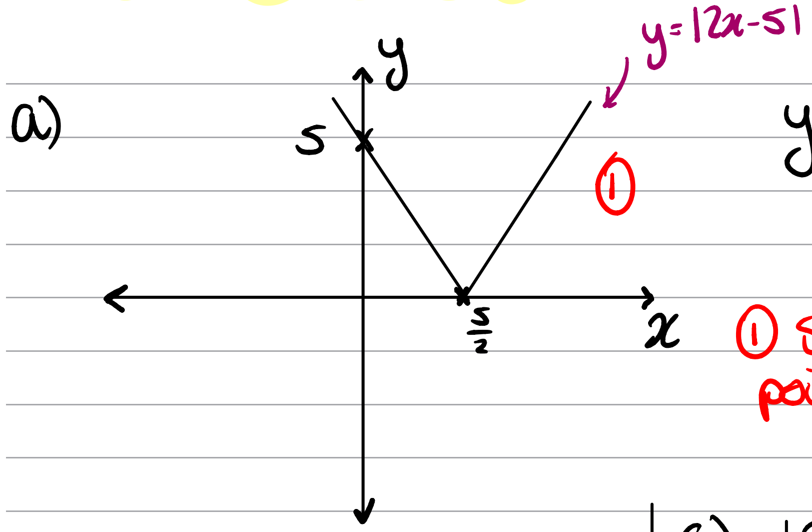
(2)

(c) Find the values of x which satisfy

$$|2x - 5| > x - \frac{5}{2}$$

Write your answer in set notation.
(2)

a)



① showing points

$y = 2x - 5$

x intercept when $y = 0$

$0 = 2x - 5$

$2x = 5$

$x = \frac{5}{2}$

b) $|2x-5| > 7$

$-(2x-5) > 7$

$-2x + 5 > 7$ ②

$2x - 5 > 7$ ①

①

① $2x > 12$

$x > 6$

①

② $5 > 7 + 2x$

$-2 > 2x$

$x < -1$

c) $|2x-5| > x - \frac{5}{2}$

$2x - 5 > x - \frac{5}{2}$ ③

$-(2x-5) > x - \frac{5}{2}$ ④

③ $2x > x + \frac{5}{2}$

① $x > \frac{5}{2}$

④ $-2x + 5 > x - \frac{5}{2}$

$5 > 3x - \frac{5}{2}$

$\frac{15}{2} > 3x$

$x < \frac{5}{2}$

①

$\{x : x < \frac{5}{2}\} \cup \{x : x > \frac{5}{2}\}$

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5. The line l has equation

$$3x - 2y = k$$

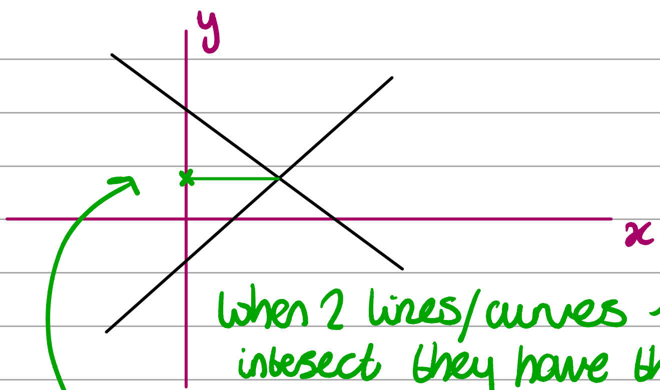
where k is a real constant.

Given that the line l intersects the curve with equation

$$y = 2x^2 - 5$$

at two distinct points, find the range of possible values for k.

(5)



When 2 lines/curves intersect they have the same 'y coordinate' so we can get where line and curve intersects by setting 'the ys' equal to each other

$3x - 2(2x^2 - 5) = k$

$3x - 4x^2 + 10 = k$

$-4x^2 + 3x + 10 - k = 0$ (2)

$b^2 - 4ac < 0$ no real roots

$b^2 - 4ac = 0$ one repeated root

$b^2 - 4ac > 0$ two real roots

Intersects at two distinct points when $b^2 - 4ac > 0$

$a = -4 \quad b = 3 \quad c = 10 - k$ (1)

$3^2 - 4(-4)(10 - k) > 0$

$9 + 16(10 - k) > 0$

$9 + 160 - 16k > 0$

$169 - 16k > 0$ (1)

$169 > 16k$

$k < \frac{169}{16}$ (1)

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6.

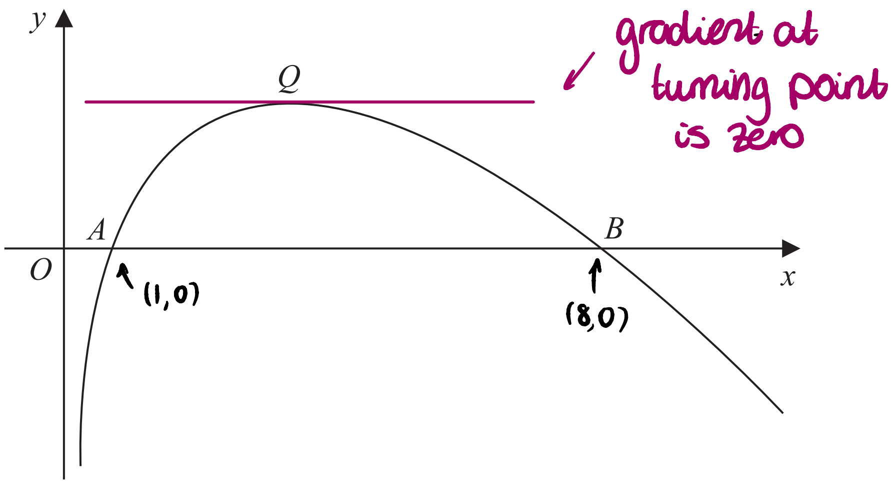


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$ $f'(x)=0$

(4)

(c) Show that the x coordinate of Q lies between 3.5 and 3.6

a b

(2)

(d) Use the iterative formula

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad n \in \mathbb{N}$$

with $x_1 = 3.5$ to

Ans

(i) find the value of x_5 to 4 decimal places,

(ii) find the x coordinate of Q accurate to 2 decimal places.

(2)

a) $0 = (8-x)\ln x$

$8 - x = 0$ $\quad$ $\ln x = 0$

$x = 8$ (1) $\quad$ $e^{\ln x} = e^0$

B $\quad$ $x = 1$

A

b) Differentiate using the product rule

$f(x) = (8-x)\ln x$

$f'(x) = (-1 \times \ln x) + \left(\frac{1}{x} \times (8-x)\right)$

$f'(x) = -\ln x + \frac{8-x}{x}$ 2

1

to find turning points
let $f'(x) = 0$

$0 = -\ln x + \frac{8-x}{x}$

$0 = -x\ln x + 8 - x$

$x\ln x + x = 8$

$x(\ln x + 1) = 8$

1

$x = \frac{8}{1+\ln x}$ as needed

c) Sign change rule - If $f(x)$ is a continuous function and a and b are numbers such that $f(x)$ changes sign between a and b, then the equation $f(x)=0$ has a root between a and b

$f'(x) = -\ln x + \frac{8-x}{x}$ $f'(3.5) = 0.03295...$ 1 $f'(3.6) = -0.05871...$

$\therefore$ Sign change and as $f'(x)$ is continuous, the x coordinate of Q lies between $x = 3.5$ and $x = 3.6$ 1

d) i) $x_5 = 3.5340$ 1

using 'Ans' button on calculator

ii) $x_Q = 3.54$ (2dp) 1

(Total for Question 6 is 9 marks)

7. A bacterial culture has area p mm^2 at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

p p

(a) Show that the scientist's model for p leads to the equation

$$p = ae^{kt}$$

where a and k are constants.

(4)

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

(b) Estimate, to 2 significant figures, the value of a and the value of k.

(3)

(c) Hence show that the model for p can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant b.

(2)

With reference to this model,

(d) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(e) State a long term limitation of the model for p.

(1)

a) $\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$ (1)

use sepeation of variables

$\frac{1}{p} \times \frac{dp}{dt} = k$

$\int \frac{1}{p}\, dp = \int k\, dt$ (1)

$\ln p = kt + c$ (1)

$e^{\ln p} = e^{kt+c}$

$p = e^{kt+c}$

using fact $x^a \times x^b = x^{a+b}$

$p = e^{kt} \times e^c$

let $e^c = a$

$p = ae^{kt}$ as needed (1)

Question 7 continued

b) $p = ae^{kt}$ (1)

$\ln p = \ln(ae^{kt})$

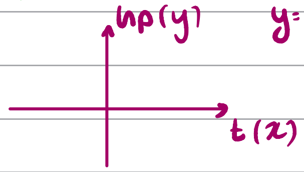
using $\ln a + \ln b = \ln(ab)$

$\ln p = \ln a + \ln e^{kt}$

$\underbrace{\ln p}_{y} = \ln a + \underbrace{kt}_{x}$

$y = mx + c$ ∴ gradient $= k$

$\ln p = kt + \ln a$ vertical intercept $= \ln a$



$y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{\text{vertical intercept}}$

(1)

$k = 0.14$

$\ln a = 3.95$

$a = e^{3.95}$

$a = 51.935$ 3dp

$= 52$ 2sf (1)

c) $p = ae^{kt}$ (1)

$p = a(e^k)^t$

let $b = e^k$

$p = ab^t$

Since $k = 0.14$ $a = 52$

$p = 52 \times \underbrace{(e^{0.14})}_{b}{}^t$ $b = e^{0.14}$

$= 1.15$ 3sf (1)

d) $p = ab^t$ (area: p, time hrs: t)

when $t = 0$

i) $p = ab^0$

$p = a(1)$

$p = a$ ∴ a represents the initial area of bacterial culture that was first placed onto the circular dish (1)

ii) Rate of increase per hour of the area of bacterial culture (1)

e) The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area. (1)

8.

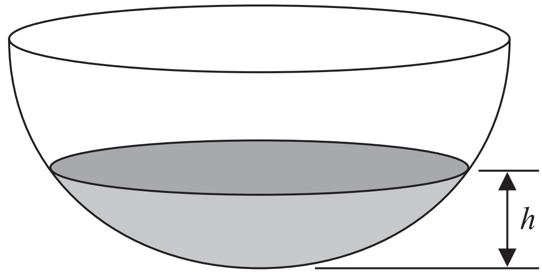


Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm^3, according to the model is given by

$$V = \frac{1}{3}\pi h^2(75 - h), \qquad 0 \leqslant h \leqslant 24 \quad \frac{dv}{dt}$$

The flow of water into the bowl is at a constant rate of 160π cm^3 s^{-1} for $0 \leqslant h \leqslant 12$

(a) Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 10$ **(5)**

$\frac{dh}{dt}$

Given that the flow of water into the bowl is increased to a constant rate of 300π cm^3 s^{-1} for $12 < h \leqslant 24$

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 20$ **(2)**

a) $\frac{dv}{dt} = 160\pi \qquad \frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi h^2(75-h)$
$= 25\pi h^2 - \frac{1}{3}\pi h^3$

? $\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$ ✓ ✓

$\frac{dV}{dh} = 50\pi h - \pi h^2$ 2

$\frac{dh}{dt} = \frac{1}{50\pi h - \pi h^2} \times 160\pi$ 1

$= \frac{160\pi}{50\pi h - \pi h^2}$

when $h = 10$ 1 $\quad \frac{dh}{dt} = \frac{160\pi}{50\pi(10) - \pi(10)^2}$

$= 0.4$ cm/s 1

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Question 8 continued

b) $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ 300π $\frac{dV}{dh} = 50\pi h - \pi h^2$

From part a

$\frac{dh}{dt} = \frac{1}{50\pi h - \pi h^2} \times 300\pi$

$= \frac{300\pi}{50\pi h - \pi h^2}$ when $h = 20$ $\frac{dh}{dt} = \frac{300\pi}{50\pi(20) - \pi(20)^2}$ 1

$= 0.5$ cm/s 1

(Total for Question 8 is 7 marks)

9. A circle with centre $A(3,-1)$ passes through the point $P(-9,8)$ and the point $Q(15,-10)$

(a) Show that PQ is a diameter of the circle.
(2)

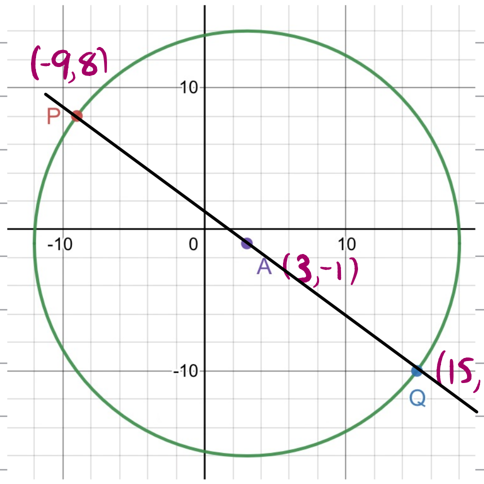
(b) Find an equation for the circle.
(3)

A point R also lies on the circle.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.
(2)

(d) Find the size of angle ARQ, giving your answer to the nearest 0.1 of a degree.
(2)

a)



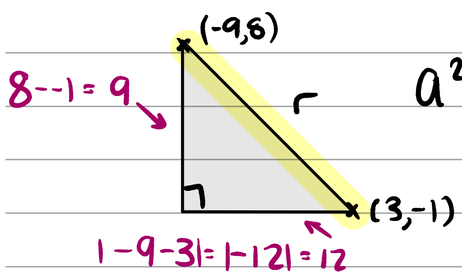
$(x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint $PQ = \left(\frac{-9+15}{2}, \frac{8-10}{2}\right)$ (1)

$= (3,-1)$ which is the center point A, so PQ is the diameter of the circle (1)

b) we need the centre point and the radius

$(3,-1)$ $r=15$



$a^2+b^2=c^2 \therefore 9^2+12^2=r^2$

$r^2=225$

$r=15$ (1)

equation of a circle $(x-a)^2+(y-b)^2=r^2$ where centre (a,b) and radius is r

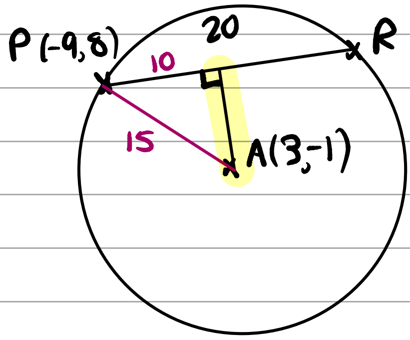
$(x-3)^2+(y+1)^2=15^2$ (1)

Question 9 continued

c) $(x-3)^2+(y+1)^2=15^2$

$P(-9,8)$

The perpendicular from the centre to a chord bisects the chord



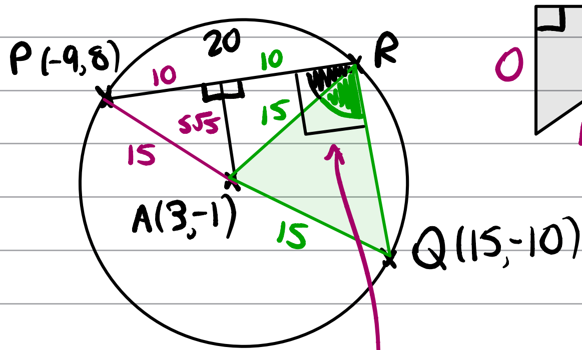
$a^2+b^2=c^2$

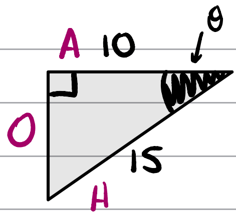
$10^2+b^2=15^2$ (1)

$b^2=125$

$b=5\sqrt{5}$ (1)

d)





SOHCAHTOA

$\cos\theta=\frac{10}{15}$

$\theta=\cos^{-1}\left(\frac{10}{15}\right)$

$=48.1897$ 4dp (1)

Angle ARQ $=90-48.1897$

$=41.8103$

$=41.8°$ (1)

(to 0.1 of a degree)

Using the fact that the angle at the circumference in a semicircle is always a right angle

(Total for Question 9 is 9 marks)

10.

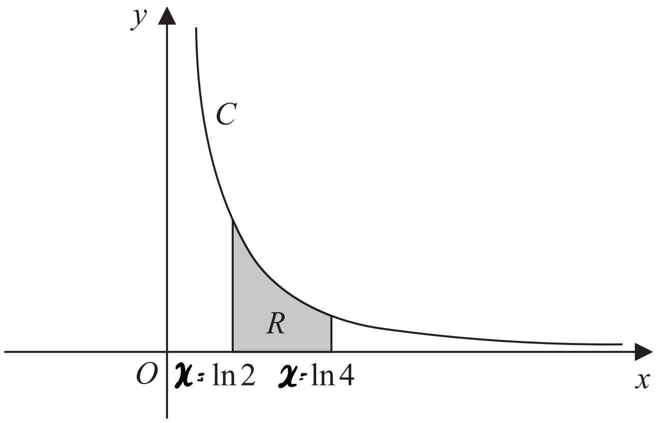


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}$$

(a) State the domain of values of x for the curve C.

(1)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = \ln 2$, the x-axis and the line with equation $x = \ln 4$

(b) Use calculus to show that the area of R is $\ln\left(\frac{3}{2}\right)$.

(8)

a) Since $t > \frac{-2}{3}$ $\quad x > \ln\left(\frac{-2}{3} + 2\right)$ $\quad \therefore \; x > \ln\left(\frac{4}{3}\right)$ ✓ (1)

b) $\int_a^b y \, dx$ instead we need to $\int_{t_1}^{t_2} y \frac{dx}{dt} \, dt$ ✓ ✓ ✓ ✓ (1)

$x = \ln(t+2)$ $\qquad y = \frac{1}{t+1}$ ✓

$\frac{dx}{dt} = \frac{1}{t+2}$ ✓ $\qquad$ Finding limits of integration

$x = \ln(t+2)$

$x = \ln(2)$ $\qquad x = \ln(4)$

$t + 2 = 2$ $\qquad t + 2 = 4$

$t_1 = 0$ ✓ $\qquad t_2 = 2$ ✓

Question 10 continued

$t_1 = 0 \quad t_2 = 2 \qquad \frac{dx}{dt} = \frac{1}{t+2} \qquad y = \frac{1}{t+1} \qquad \int_{t_1}^{t_2} y \frac{dx}{dt}\,dt$

$$\int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt \quad 1$$

use partial fractions to integrate

$$\frac{1}{(t+1)(t+2)} = \frac{A}{(t+1)} + \frac{B}{(t+2)} \quad 1$$

$$1 = A(t+2) + B(t+1)$$

let $t=-2$ $\quad 1 = -B$

$B = -1$

let $t = -1$ $\quad 1 = A$

$A = 1$

$$\therefore \frac{1}{(t+1)(t+2)} = \frac{1}{t+1} - \frac{1}{t+2} \quad 1$$

$$\int_0^2 \frac{1}{t+1} - \frac{1}{t+2}\,dt$$

using $\int \frac{f'(x)}{f(x)} = \ln|f(x)| + c$

2

$$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$$

$$= [\ln(3) - \ln(4)] - [\underbrace{\ln(1)}_{0} - \ln(2)] \quad 1$$

$$= \ln 3 - \ln 4 + \ln 2$$

$$= \ln\left(\frac{3\times 2}{4}\right)$$

$$= \ln\left(\frac{6}{4}\right) \quad 1$$

$$= \ln\left(\frac{3}{2}\right) \text{ as needed}$$

(Total for Question 10 is 9 marks)

11. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(5)

$T_2 = 2k \quad T_3 = 5k - 10 \quad T_4 = 7k - 14$

$T_4 - T_3 = d$

common difference

$T_3 - T_2 = d$

$(7k-14) - (5k-10) = 2k - 4 = d$ ① $\quad \therefore 2k - 4 = 3k - 10$

$(5k-10) - (2k) = 3k - 10 = d \quad -4 = k - 10$

$k = 6$ ①

using $k=6$ we can work out first term and common difference

$d = 2(6) - 4 = 8$ ①

$a = T_2 - 8 = 2(6) - 8 = 4$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$S_n = \frac{1}{2}n[2(4) + (n-1)(8)]$ ①

$= 4n + \frac{1}{2}n(n-1)(8)$

$= 4n + 4n(n-1)$

$= \cancel{4n} + 4n^2 - \cancel{4n}$

$= 4n^2 = (2n)^2$ which is a square number as needed

①

12. A curve C is given by the equation

$$\sin x + \cos y = 0.5 \qquad -\frac{\pi}{2} \leqslant x < \frac{3\pi}{2}, -\pi < y < \pi$$

A point P lies on C.

The tangent to C at the point P is parallel to the x-axis.

$\frac{dy}{dx} = 0$

Find the exact coordinates of all possible points P, justifying your answer.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

$\sin x + \cos y = 0.5$

Implicit differentiation

(1)

$\cos x - \sin y \frac{dy}{dx} = 0$

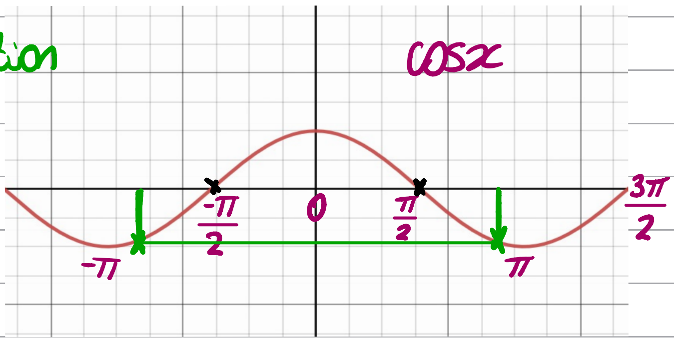
$\frac{dy}{dx} = 0$

$\cos x - \sin y(0) = 0$

$\cos x = 0$ (1)

$x = \cos^{-1}(0)$

$= \frac{\pi}{2}, \frac{-\pi}{2}$ (1)



Use curve to find all the possible solutions

$\cos y = 0.5 - \sin x$

let $x = \frac{\pi}{2} \Rightarrow \cos y = 0.5 - 1 = \frac{-1}{2}$

(1) Correct process

$y = \cos^{-1}\left(\frac{-1}{2}\right)$

$= \frac{2\pi}{3}, \frac{-2\pi}{3}$ (1)

$\therefore$ In specified range (1)

$(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, \frac{-2\pi}{3}\right)$

let $x = \frac{-\pi}{2} \Rightarrow \cos y = 0.5 + 1$

$= 1.5$

$y = \cos^{-1}(1.5)$

$=$ No solution

because $\cos y = 1.5$ has no solutions there are exactly 2 possible points for P (1)

S 6 0 7 3 7 A 0 2 6 3 2

13. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x, \qquad x \neq 90n°, n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

a) LHS $= \operatorname{cosec} 2x + \cot 2x$

$\operatorname{cosec} x = \dfrac{1}{\sin x}$

$\cot x = \dfrac{\cos x}{\sin x}$

$= \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$ 1

$= \dfrac{1 + \cos 2x}{\sin 2x}$ 1

$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

$= \dfrac{\cancel{1} + 2\cos^2 x \cancel{- 1}}{2\sin x \cos x}$ 2

$= \dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x \cancel{\cos} x}$

$= \dfrac{\cos x}{\sin x} = \cot x = \text{RHS}$ 1

b) $2x = 4\theta + 10°$
$x = 2\theta + 5°$

$\cot(2\theta + 5°) = \sqrt{3}$ 1

$\dfrac{1}{\tan(2\theta + 5°)} = \sqrt{3}$

$0 \leqslant \theta < 180°$

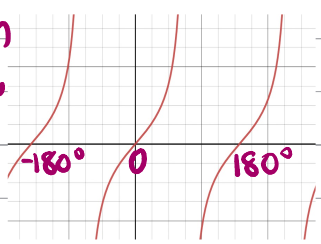
$\tan(2\theta + 5°) = \dfrac{1}{\sqrt{3}}$

$5° \leqslant 2\theta + 5° < 365°$ ★

★ $2\theta + 5° = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$2\theta + 5° = 30°, 210°$

tan



use fact that tan repeats ever 180°

$2\theta + 5° = 30°$
$2\theta = 25°$
$\theta = 12.5°$ 2

$2\theta + 5° = 210°$
$2\theta = 205°$
$\theta = 102.5°$ 2

S 6 0 7 3 7 A 0 2 8 3 2

14. (i) Kayden claims that

$$3^x \geqslant 2^x$$

Determine whether Kayden's claim is ~~always true~~, sometimes true or ~~never true~~, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number. Proof by contradiction

(6)

i) let $x = 2$ $\quad 3^2 = 9 \quad 2^2 = 4 \quad 9 \geqslant 4 \;\therefore\; 3^x \geqslant 2^x$ when $x = 2$ (1)

let $x = -1$ $\quad 3^{-1} = \frac{1}{3} \quad 2^{-1} = \frac{1}{2} \quad \frac{1}{3} \leqslant \frac{1}{2} \;\therefore\; 3^x \leqslant 2^x$ when $x = -1$ (1)

so the claim $3^x \geqslant 2^x$ is sometimes true

ii) Assume $\sqrt{3}$ is rational (1)

$\sqrt{3} = \frac{p}{q}$ where p, q are integers and $\frac{p}{q}$ is in simplest form

$3 = \frac{p^2}{q^2} \Rightarrow 3q^2 = p^2$ (2) p^2 is divisible by 3 and so p is divisible by 3

let $p = 3k$ where k is an integer (2)

$3q^2 = (3k)^2 \Rightarrow 3q^2 = 9k^2 \Rightarrow q^2 = 3k^2$

q^2 is divisible by 3 and so q is divisible by 3

eg $\frac{\cancel{3}a}{\cancel{3}b} = \frac{a}{b}$

As both p and q are divisible by 3 then $\frac{p}{q}$ can't have been in its simplest form (1)

$\therefore$ This contradiction implies that $\sqrt{3}$ is irrational